

BELLCOMM, INC.

1100 Seventeenth Street, N.W. Washington, D.C. 20036

SUBJECT: Interaction of Space Probes
with Planetary Atmospheres:
II - Case 710

DATE: November 16, 1967

FROM: R. N. Kostoff

ABSTRACT

Closed-form expressions are obtained for both drag forces experienced by, and kinetic energy lost by, a manned planetary flyby vehicle due to its interaction with the planetary atmosphere.

The major simplification in the derivation of the energy loss formula is the replacement of the hyperbolic flyby trajectory near periapsis by a circular trajectory. Errors arising from this assumption of a circular trajectory are examined.

In situ measurement of density at periapsis is examined, and is shown to be desirable.

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MEMORANDUM FOR FILEINTRODUCTION

In a manned planetary flyby mission, it is desirable to have the flyby vehicle approach the planet to within reasonably close distances. However, the closer the vehicle comes to the planet, the more important becomes the presence of the atmosphere. Uncertainties in knowledge of the state of the upper atmosphere before flyby result in uncertainties in calculation of both the drag forces which will act on the vehicle and kinetic energy lost by the vehicle through dissipation. The following analysis will show explicitly the dependence of drag force and dissipated energy on the atmospheric parameters, and will allow an estimate of the effect of uncertainties in knowledge of the atmospheric parameters on uncertainties in drag force and energy dissipation calculations.

NOMENCLATURE

The symbols are listed in approximate order of usage.

- E_D - kinetic energy dissipated by manned vehicle along flyby trajectory
- $F_D(r)$ - non-electromagnetic drag force acting on vehicle
- S - length coordinate along vehicle trajectory
- r - polar coordinate of length, from center of planet to point on trajectory
- θ - polar angular coordinate
- C_D - free-molecule drag coefficient of flyby vehicle
- $\rho(r)$ - atmospheric density
- A - cross-sectional area of vehicle
- $V(r)$ - vehicle velocity along flyby trajectory

- ρ_s - atmospheric density at planet surface
- r_p - planet radius
- H - atmospheric scale height
- V_∞ - velocity of vehicle relative to the planet when the vehicle is outside the planet's gravitational field
- g_0 - gravitational acceleration at planet's surface
- C, θ' - trajectory constants
- ϵ - eccentricity of orbit
- ψ - angular polar coordinate, measured from line joining periapsis to center of planet
- r_0 - distance from center of planet to periapsis point
- Y - angular polar coordinate
- ψ_{\max} - upper limit of ψ used for evaluating integral
- n - number of scale heights between r and r_0 when $\psi = \psi$
- ϕ - turning angle of trajectory

ANALYSIS

Figure 1 shows the flyby trajectory. To obtain the total vehicle kinetic energy dissipated along the trajectory (neglecting electromagnetic drag, solar radiation drag, etc.) due to vehicle-planetary atmosphere interaction, the following integral must be evaluated:

$$E_D = \int_S F_D(r, \theta) \cdot dS \quad (1)$$

where

E_D is the total vehicle kinetic energy which is dissipated by the vehicle-atmosphere interaction;

$F_D(r, \theta)$ is the atmospheric drag force which acts on the vehicle at a point (r, θ) on the trajectory;

S is a length coordinate which is measured along the trajectory.

If $S = 0$ denotes periapsis, then the integral extends from $S = -\infty$ to $+\infty$. As will be shown later, it will not be necessary to carry the integration past relatively small values of S .

In polar coordinates dS , a differential element of length along the trajectory, takes the form:

$$dS = \left[r^2 + \left(\frac{dr}{d\theta} \right)^2 \right]^{1/2} d\theta \quad (2)$$

where r and θ are shown in Figure 1.

$F_D(r, \theta)$ may be written in the usual manner:⁽¹⁾

$$F_D(r, \theta) = C_D \cdot 1/2 \rho(r) A V^2(r, \theta) \quad (3)$$

where

C_D is the drag coefficient of the flyby vehicle. Due to the fact that periapsis altitude will be of the order of a few hundred kilometers, (where the appropriate⁽²⁾ mean free path is large compared to a relevant vehicle dimension), the drag coefficient will assume its free molecule value.⁽³⁾

$\rho(r)$ is the atmospheric density at distance r from the planet center; A is the mean cross-sectional area of the vehicle which is normal to the vehicle velocity vector;

$V(r, \theta)$ is the vehicle velocity at point r .

$\rho(r)$ may be assumed to be of the form: (4)

$$\rho(r) = \rho_s e^{-\frac{r-r_p}{H}} \quad (4)$$

where

ρ_s is the atmospheric density at the planet surface;

r_p is the planet radius.

H is the atmospheric scale height. For convenience in calculations, it is assumed to be constant from the planet surface to the flyby vehicle trajectory. Figure 2 is a plot of number density vs. altitude for a collection of postulated Martian atmospheres⁽⁵⁾. Added to the graphs contained in Reference 5 is a graph of the model "F" Martian atmosphere. This atmosphere has the properties contained in Table 1. However, it's forte is a constant scale height throughout the atmosphere. As Figure 2 shows, the model "F" atmosphere approximates quite well that postulated by Chamberlain and McElroy. It is directly applicable to the present analysis, where a constant scale height atmosphere is specified.

$V^2(r, \theta)$, for the specific case of a vehicle which is within the gravitational field of the planet, may be written as: (6)

$$V^2(r, \theta) = V_\infty^2 + \frac{2g_0 r_p^2}{r} \quad (5)$$

where

V_∞ is the velocity of the vehicle, relative to the planet velocity, before it is perturbed by the planet's gravitational field, and g_0 is the gravitational acceleration existing at the planet's surface.

Combination of equations (1), (2), (3), (4), and (5) produces the following expression for E_D

$$E_D = \int_{\theta, r} C_D \cdot 1/2 \rho_s e^{-\frac{r-r_p}{H}} A(rV_\infty^2 + 2g_0 r_p^2) \left[1 + \left(\frac{1}{r} \frac{dr}{d\theta} \right)^2 \right]^{1/2} d\theta \quad (6)$$

To readily evaluate the integral, the integrand must be simplified.

r is related to θ through the defining equation for the hyperbolic flyby trajectory: ⁽⁷⁾

$$\frac{1}{r} = C \left[1 + \epsilon \cos(\theta - \theta') \right] \quad (7)$$

where

C, θ' are constants of the trajectory;

ϵ is the eccentricity of the orbit, and is greater than unity for a hyperbola.

Figure 1 shows that $(\theta - \theta')$ is the polar coordinate angle measured from the line joining the periapsis point with the center of the planet. The following substitution is now made:

$$\psi = \theta - \theta' \quad (8)$$

When $\psi = 0$ (periapsis), $r = r_0$ (distance from the periapsis point to the planet center). Insertion of this boundary condition into equation (7) gives, for C :

$$C = \frac{1}{r_0(1+\epsilon)} \quad (9)$$

Substitution of equations (8) and (9) into (7) yields:

$$\frac{1}{r} = \frac{1}{r_0(1+\epsilon)} \left[1 + \epsilon \cos \psi \right] \quad (10)$$

Differentiation of equation (10) with respect to ψ produces the following:

$$\frac{1}{r} \frac{dr}{d\psi} = \frac{\epsilon \sin \psi}{1+\epsilon \cos \psi} \quad (11)$$

ψ is now assumed to be no greater than 15° , so that $\sin \psi$ may be replaced by ψ and $\cos \psi$ may be replaced by $1 - \frac{\psi^2}{2}$, with an attendant error of 1% or less. $\frac{1}{r} \frac{dr}{d\psi}$ increases in magnitude with increasing ϵ . The maximum value of $\frac{1}{r} \frac{dr}{d\psi}$ ($\epsilon \gg 1$) is given by the following expression:

$$\left(\frac{1}{r} \frac{dr}{d\psi} \right)_{\max} = \tan \psi \sim \psi \quad (\text{for small } \psi) \quad (12)$$

In equation (6) the maximum value of the term $\left[1 + \left(\frac{1}{r} \frac{dr}{d\psi} \right)^2 \right]^{1/2}$, substituting equation (12), becomes:

$$\left[1 + \left(\frac{1}{r} \frac{dr}{d\psi} \right)^2 \right]_{\max}^{1/2} = (1+\psi^2)^{1/2} \quad (13)$$

which, for $\psi \ll 1$, is:

$$(1+\psi^2)^{1/2} = 1 + \frac{\psi^2}{2} \quad (14)$$

Thus, the right hand side of equation (14) may be replaced by unity with an error of approximately $\frac{\psi^2}{2}$. In the present case, this error is no greater than 3%. Physically, the result of this approximation is that the segment of the hypernolic trajectory near periapsis ($\psi < 15^\circ$) may be replaced by a circular segment with an error of no more than 3%. Mathematically, it means:

$$\left[r^2 + \left(\frac{dr}{d\theta} \right)^2 \right]^{1/2} \sim r. \quad (15)$$

The next simplification of the integrand in (6) is the replacement of the variable coefficient of V_∞^2 , r , by some average value of r in the interval of interest. In this interval, r ranges from r_0 (its minimum value, at periapsis) to $r_0 + nH$ (its maximum value). As will be shown later, the integral (6) may be truncated at some relatively small value of ψ . When ψ equals this limiting value, r equals its maximum value in this interval, $r_0 + nH$. n denotes the number of scale heights above periapsis by which r is increased. Thus, r may be replaced in the interval of interest by its average value $r_0 + \frac{nH}{2}$, with an attendant error of at most $\frac{\frac{nH}{2}}{r_0 + \frac{nH}{2}}$.

Later results (which obtain the value of n) will show that this error will be less than 1% for Mars (where $H \sim 10$ km and $r_0 \sim 3700$ km).

However, the small variation in r in the range $0 < \psi < 15^\circ$ produces a much more significant change in the $e^{-\frac{r}{H}}$ term in equation (6). $e^{-\frac{r}{H}}$ may be written, with the aid of equation (10), as:

$$e^{-\frac{r}{H}} = e^{-\frac{r_0}{H}} \frac{(1+\epsilon)}{(1+\epsilon \cos \psi)} \quad (16)$$

Utilization of the small angle assumption ($\cos \psi \approx 1 - \frac{\psi^2}{2}$) transforms equation (16) into:

$$e^{-\frac{r}{H}} = e^{-\frac{r_0}{H}} e^{-\frac{r_0}{H}} \frac{\epsilon}{2(1+\epsilon)} \psi^2 \quad (17)$$

A change of variables will simplify (16) somewhat. Let:

$$Y^2 = \frac{r_0}{H} \frac{\epsilon}{2(1+\epsilon)} \psi^2 \quad (18)$$

with

$$dY = \sqrt{\frac{r_0}{H} \frac{\epsilon}{2(1+\epsilon)}} d\psi \quad (19)$$

Combination of equations (17) and (18) gives:

$$e^{-\frac{r}{H}} = e^{-\frac{r_0}{H}} e^{-Y^2} \quad (20)$$

Insertion of equations (15), (19) and (20) into equation (6), with replacement of r by $r_0 + \frac{n}{2}H$, produces:

$$E_D = \int_{Y(\psi)} C_D \cdot 1/2 \rho_s A \left[\left(r_0 + \frac{n}{2}H\right) V_\infty^2 + 2g_0 r_p^2 \right] e^{-\frac{r_0 - r_p}{H}} \left(\frac{H}{r_0} \frac{2(1+\epsilon)}{\epsilon} \right)^{1/2} e^{-Y^2} dY \quad (21)$$

At this point the limits of integration must be established. Due to symmetry of the flyby trajectory about periapsis, the integration may be performed from $\psi = 0$ (periapsis) to $\psi = \psi_{\max}$ (an arbitrarily selected value of ψ above which the contribution to the integral is deemed negligible), and the result multiplied by two. This is equivalent to the integral from $\psi = -\psi_{\max}$ to $\psi = \psi_{\max}$. ψ_{\max} will now be obtained by integrating equation (21) to its variable upper limit (ψ_{\max}), then permitting this angular limit to increase until the magnitude of the

integral approaches arbitrarily closely a definite limiting value. When $\psi = \psi_{\max}$, $r = r_o + nH$, where n must be determined. Substitution of $r_o + nH$ in equation (10), and utilization of the assumptions that ψ is a small angle ($\cos \psi \sim 1 - \frac{\psi^2}{2}$), and that $nH \ll r_o$ (quasi-circular trajectory assumption), gives the following:

$$\psi_{\max} = \left[\frac{2nH}{r_o} \frac{(1+\epsilon)}{\epsilon} \right]^{1/2} \quad (22)$$

Combination of equations (18) and (22) yields:

$$Y_{\max} = \sqrt{n} \quad (23)$$

Integration of equation (21) from $Y = 0$ to $Y = \sqrt{n}$, and multiplication of the result by two, produces:

$$E_D = C_D \rho_s A \left[(r_o + \frac{nH}{2}) V_{\infty}^2 + 2g_o r_p^2 \right] e^{-\frac{r_o - r_p}{H}} \left(\frac{H}{r_o} \frac{2(1+\epsilon)}{\epsilon} \right)^{1/2} \sqrt{\frac{\pi}{2}} \operatorname{erf} \sqrt{n} \quad (24)$$

where

$$\int_0^{\sqrt{n}} e^{-Y^2} dY = \sqrt{\frac{\pi}{2}} \operatorname{erf} \sqrt{n} \quad (25)$$

Figure 3, a plot of $\operatorname{erf} \sqrt{n}$ vs. n , shows that $\operatorname{erf} \sqrt{n}$ approaches within 1% of its limiting value of unity when $n \sim 3$. Thus, the upper limit of integration is ψ_{\max} corresponding to $r = r_o + 3H$. Equation (22) shows ψ_{\max} to range from 11° ($\epsilon \sim 1$) to 8° ($\epsilon \sim \infty$).

Replacement of $\operatorname{erf} \sqrt{n}$ by unity in equation (24) gives the final expression for E_D :

$$E_D = C_D A \rho_s e^{-\frac{r_o - r_p}{H}} \left[(r_o + \frac{3}{2}H) V_\infty^2 + 2g_o r_p^2 \right] \left[\frac{H}{r_o} \frac{2(1+\epsilon)}{\epsilon} \right]^{1/2} \quad (26)$$

The maximum value of F_D (when $r = r_o$) is (equation [3]):

$$F_{D \max} = C_D \cdot 1/2 \rho_s e^{-\frac{r_o - r_p}{H}} A \left(V_\infty^2 + \frac{2g_o r_p^2}{r_o} \right) \quad (27)$$

SUMMARY AND CONCLUSIONS

A formula (26) for predicting vehicle kinetic energy loss due to interaction with the atmosphere has been obtained. Now, some of the assumptions used in deriving the formula will be discussed. Also, a physical description of the formula will be given.

The major assumption that $\psi < 15^\circ$ is shown to be valid by the result that the integral reaches a limiting value for $\psi_{\max} \sim 11^\circ$. Thus, the largest error due directly to this small angle assumption, which is the second term on the right-hand side of equation (14), $\frac{\psi^2}{2}$, is seen to be about 2%.

E_D , as given in equation (26) may be considered as the product of drag force at periapsis (maximum drag force) times a distance X . Figure 4 shows the relation between r_o , H , and X . A triangle is constructed with two vertices located on the flyby trajectory, and the third located at the planet center.

If $\tan^{-1} \left[\frac{X}{r_o} \right]$ is small, as is the case in the present analysis, then $X \sim \sqrt{2Hr_o}$. Thus, the total kinetic energy dissipated during flyby may be considered as the result of

maximum F_D (equation [27]) acting thru the distance from periapsis to a trajectory point of radial location one scale height above periapsis (symmetrical about periapsis, of course). The factor $\frac{1+\epsilon}{\epsilon}$ in equation (26) indicates how rapidly the trajectory 'bends away' from a circular trajectory in the near-periapsis region and thus provides a measure of the total distance traversed by the vehicle in going from r_0 to $r_0 + H$.

As equation (26) shows, E_D is approximately proportional to periapsis atmospheric density, $\rho_s e^{-\frac{r_0-r_p}{H}}$. Therefore, the uncertainty in calculation of E_D is proportional to the uncertainty of knowledge of density at periapsis. If this density is measured directly by an in situ probe before flyby, then the uncertainty in E_D , $\frac{dE_D}{E_D}$, will be directly proportional to uncertainty in density. However, if scale height at lower altitudes is measured before flyby (occultation experiment), and density at periapsis is obtained from extrapolation of density at low altitudes, then it can be shown⁽⁷⁾ that $\frac{dE_D}{E_D}$ is proportional to $\frac{(r_p-r_0)}{H} \frac{dH}{H}$, where $\frac{dH}{H}$ is the uncertainty in measurement of scale height, and $\frac{(r_p-r_0)}{H}$ is a relatively large quantity (~ 30). Thus, an in situ measurement of periapsis density⁽⁸⁾ appears quite attractive for the present application.

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Attachments

Figures 1-4

Table I

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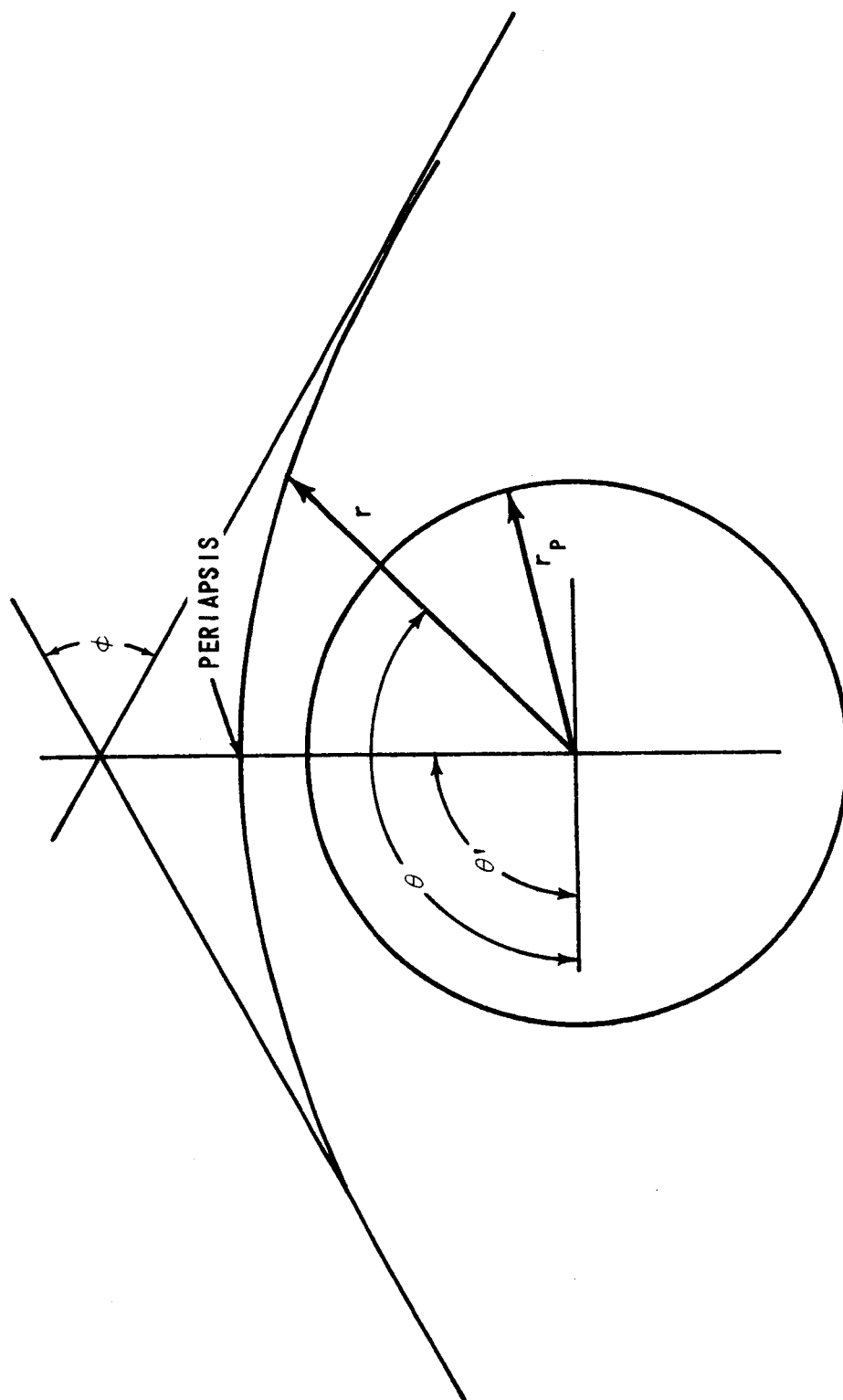


FIGURE I - ACTUAL FLYBY TRAJECTORY

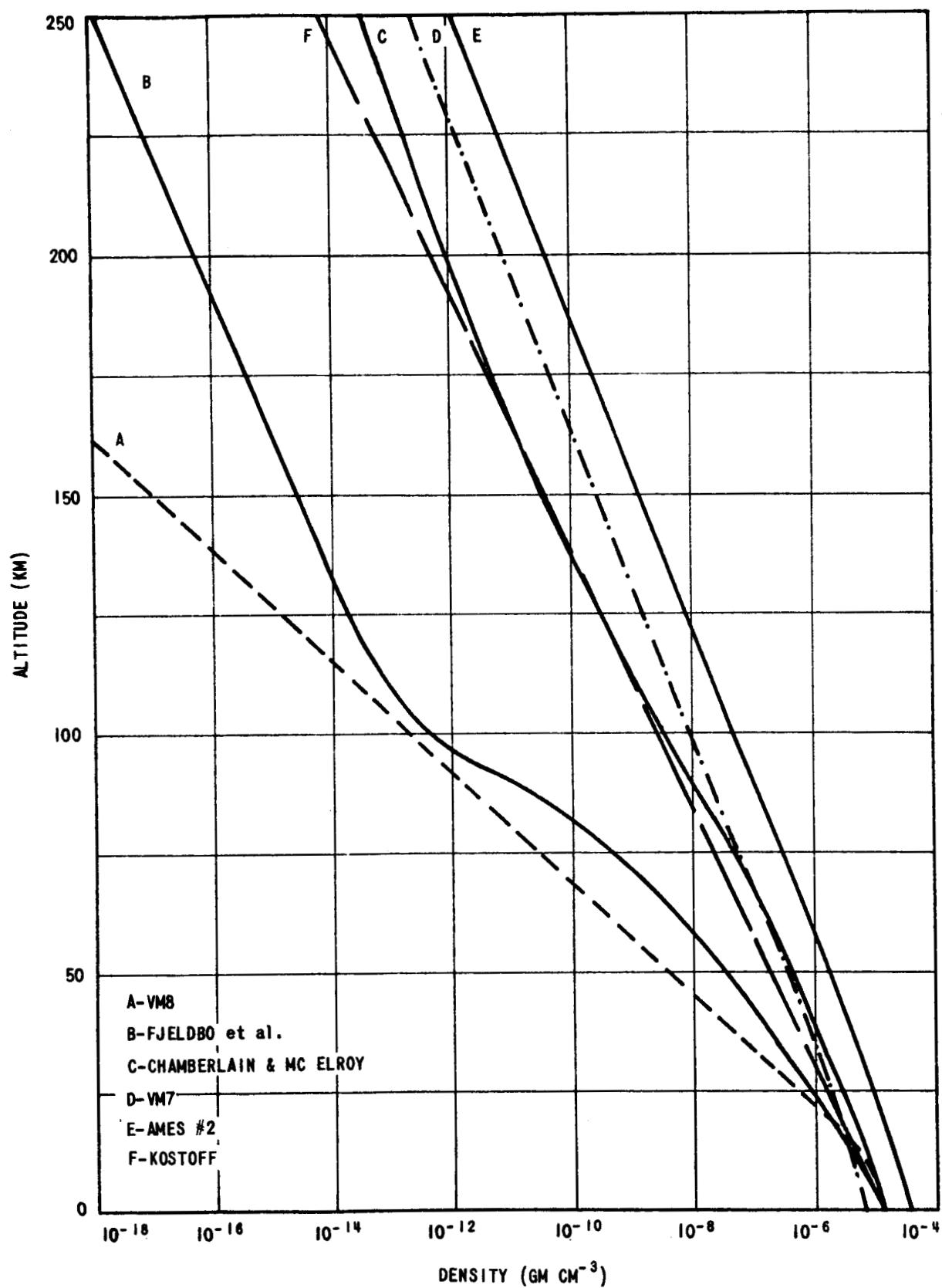
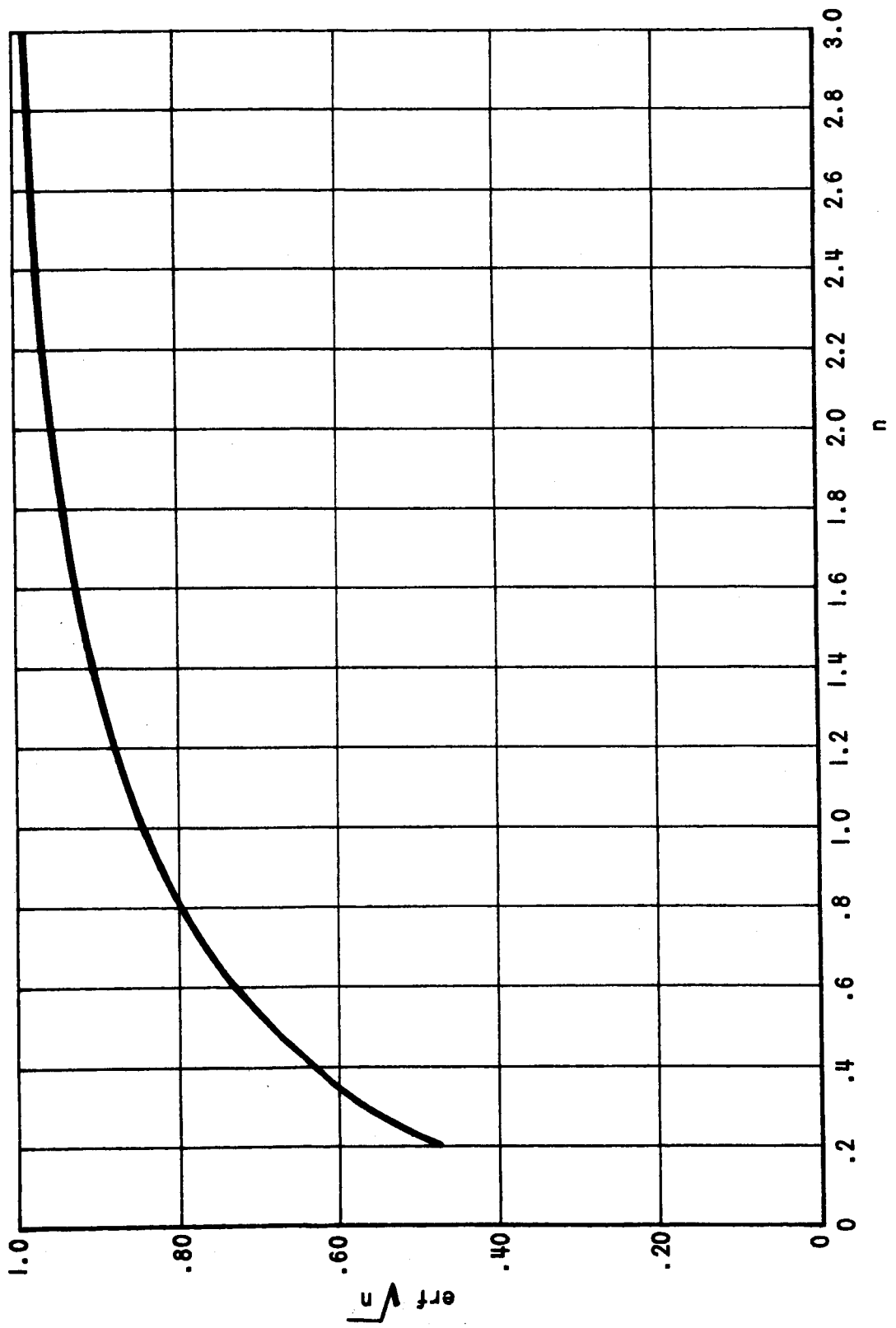


FIGURE 2 - DENSITY PROFILE IN THE MARTIAN ATMOSPHERE

FIGURE 3
 $\text{erf } \sqrt{n} \text{ vs. } n$



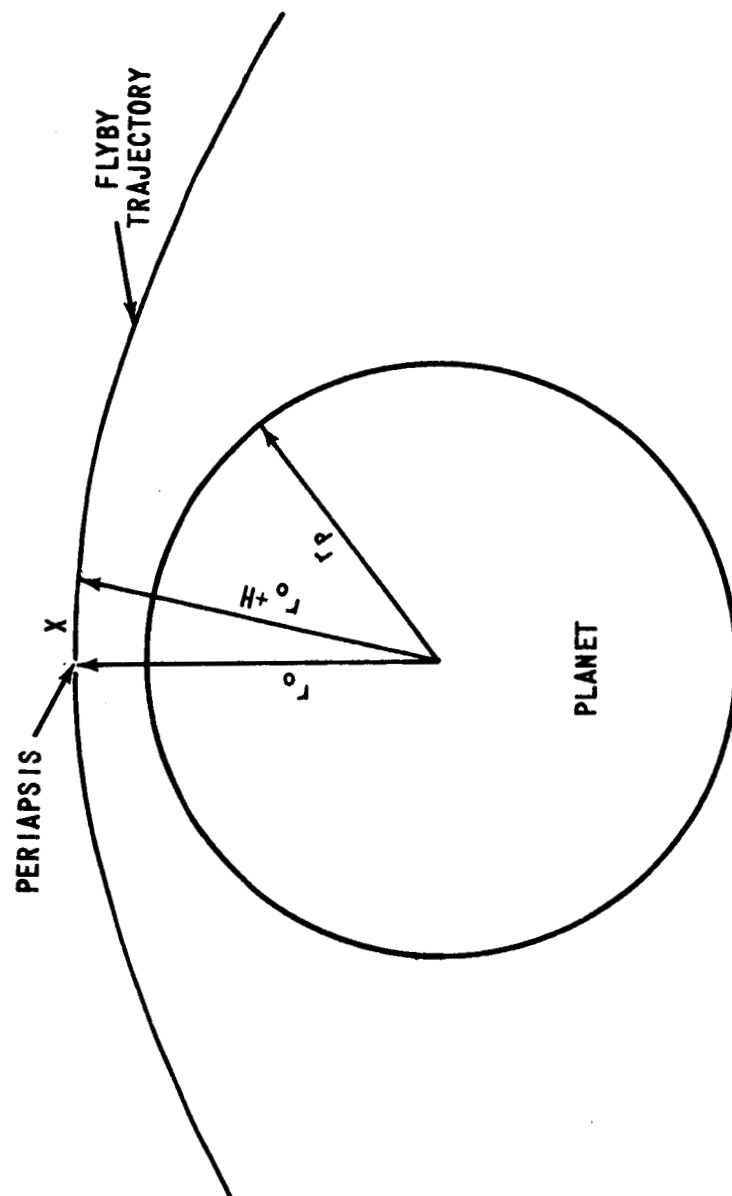


FIGURE 4

TABLE I
MODEL "F" ATMOSPHERE FOR MARS

SYMBOL	DEFINITION	VALUE	UNITS
ρ_s	SURFACE DENSITY	3.10^{-5}	SLUGS/FT ³
P_s	SURFACE PRESSURE	13.5	LBS/FT ²
H	SCALE HEIGHT	36,000	FT
R	GAS CONSTANT	1140	FT-LBS SLUG-MOLE-°R
M	MOL. WT. OF ATMOSPHERE	44	GRAMS/MOLE
T_o	SURFACE TEMPERATURE	235	°K

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Subject: Interaction of Space Probes with
Planetary Atmospheres: II

From: R. N. Kostoff

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